ACM 106C: Problem set 6

Due: May 22, 2008

1. Consider the following boundary value problem (D)

$$-u_{xx} + 4u = 12x, \quad 0 < x < 1,$$
$$u(0) = u(1) = 0.$$

We define the following: $f(x) = 12x$,

$$V = \Big\{ v \mid v \text{ is continuous on} [0,1], v' \text{ is piecewise continuous and bounded on } [0,1], \text{ and } v(0) = v(1) = 0 \Big\},$$

$$a(v, w) = \int_0^1 (v'(x)w'(x) + 4\, v(x)w(x))\, dx.$$

$$(v, w) = \int_0^1 v(x)w(x)\, dx.$$

The variational problem (V) is: find $u \in V$ such that

$$a(u, v) = (f, v), \quad \forall\, v \in V.$$

The aim is to find an approximation u_h to the above boundary value problem (D) by the piecewise linear FEM.

(a) Write down the finite dimensional problem (V_h) corresponding to the problem (V). Moreover, give the definition of the finite dimensional subspace V_h.

(b) Write down the linear system $A\xi = b$ resulting from the piecewise linear FEM for the above problem (D). Moreover, find the value of each entry of the matrix A.

(c) Find u_h by using $h = 1/20$. Plot both the exact solution u and the finite element approximation u_h on the same graph. Turn in a printout of the graph and the MATLAB M-file.

2. Prove that the matrix A in Question 1(b) is invertible.

3. Consider Question 1. Show that

$$a(u - u_h, v) = 0, \quad \forall\, v \in V_h.$$