ACM 106a, Problem Set 3

(modified November 13, 2007; see ~~strikeouts~~ for changes.)

Due: Thursday, November 15, 2007

Theory

1. Do Stoer & Bulirsch, exercise 2.31(a),(b) (pp. 142–43). Also, for certain λ_i, the sinh and cosh terms can be expressed in terms of the ordinary trigonometric functions sin and cos. What ~~is this~~ are these λ_i?

2. Stoer & Bulirsch, exercise 3.18 (p. 187).

MATLAB

1. In the first theory problem above, you showed that *exponential splines* can be written in terms of sin and cos for a certain choice of λ_i. This makes them very good for interpolating curves containing circular arcs. To show this, do the following:

   (a) ~~Sample the half-circle $f(x) = \sqrt{1 - x^2}$ at the 21 points $x = -1, -0.9, \ldots, 0.9, 1$.~~
   Sample the circle $F(t) = (\cos(t), \sin(t))$ at the 21 equally spaced points $t = -\pi, \ldots, \pi$.

   (b) Construct two interpolating splines for this function, using the special value of λ_i and using $\lambda_i = 0$.

   (c) ~~On a fine grid (201 points, $x = -1, -0.99, \ldots, 0.99, 1$), graph the two splines and plot their errors compared to $f(x)$.~~
   On a fine grid (201 equally spaced points, $t = -\pi, \ldots, \pi$), graph the two splines $(x(t), y(t)) = S(t)$, and plot their error distances $|F(t) - S(t)|$ vs. t.

   NOTE: If you have already done the original (much harder) problem of interpolating the function $f(x) = \sqrt{1 - x^2}$ with exponential splines, you may hand this in as well for extra credit.