ACM 105: Problem Set 5

Due: May 16, 2006

1. If E_1 and E_2 are measurable, show that $|E_1 \cup E_2| + |E_1 \cap E_2| = |E_1| + |E_2|$.

2. Let $\{E_k\}$ be a sequence of disjoint measurable sets and let A be any set. Show that
$$|A \cap (\cup_{k=1}^{\infty} E_k)|_e = \sum_{k=1}^{\infty} |A \cap E_k|_e.$$

3. Let E be measurable. Show that $f : E \to \bar{R}$ is measurable if and only if $\{a < f < \infty\}$ is measurable for any finite a.

4. Let $\{f_k\}$ be a sequence of measurble functions defined on a measurable set E with $|E| < +\infty$. If, for each $x \in E$, $|f_k(x)| \leq M_x < +\infty$ for all k, show that given $\epsilon > 0$, there is a closed $F \subset E$ and a finite number M such that $|E - F| < \epsilon$ and $|f_k(x)| \leq M$ for all k and for all $x \in F$.