ACM 101a November 21, 2007

Problem Set VII

1.- We say that the point z_0 is a saddle point of order N for the exponent function ϕ in the integral

$$I(k) = \int_C f(t) e^{k\phi(z)} dz$$

if the first N derivatives of ϕ vanish at $z = z_0$, and the $N+1$-th derivative at $z = z_0$ is not zero. Show that from a saddle point $z = z_0$ of order N there emanate $N+1$ directions of steepest descent and $N+1$ directions of steepest ascent, given by

$$\theta = -\alpha/(N+1) + (2m+1)\frac{\pi}{n}, \quad m = 0, 1, \ldots, N$$

and

$$\theta = -\alpha/(N+1) + 2m\frac{\pi}{n}, \quad m = 0, 1, \ldots, N$$

respectively, where θ is the polar angle around $z = z_0$, and where

$$\alpha = \arg\left(\frac{d^{N+1}\phi}{dz^{N+1}}(z_0)\right).$$

2.- Use the method of steepest descent to find the leading asymptotic behavior as $k \to \infty$ of

$$\int_{-\infty}^{\infty} \frac{t e^{ik\left(\frac{t^3}{3}+t\right)}}{1+t^4} dt.$$

3.- Show that for $0 < \theta < \pi/2$

$$\int_0^\theta e^{-k\sec x} dx \sim \sqrt{\frac{\pi}{2k}} e^{-k}, \quad k \to \infty.$$

Due November 28 at noon.