ACM 101a October 10, 2007

Problem Set II

1.- Evaluate

$$\int \frac{dz}{(1+z^2)^{1/2}}$$

from $z = -2$ to $z = +2$ around the semicircle $|z| = 2$ in the upper half plane. (Complete the definition of the function $(1 + z^2)^{1/2}$ by choosing as branch line the part of the imaginary axis satisfying $|y| \leq 1$ and enforcing that, for $z = -1$, $(1 + z^2)^{1/2}$ equals the negative square root of 2.)

2.- Explain why it is valid to obtain the series in powers of z for for the function $\arcsin z$ by integrating the expansion of $(1 - z^2)^{-1/2}$ term by term. Discuss the choice of branch.

3.-

(a) Show that formal term-by-term differentiation or integration of a power series yields a new power series with the same radius of convergence.

(b) The uniform-convergence property of a power series implies that term by term integration yields the integral of the sum function. Show that the integrated sum function is single-valued and analytic within the circle of convergence.

(c) Show that a power series converges to an analytic function within its circle of convergence.

(d) In class we have evaluated derivatives of integrals by exchanging the differentiation and integration processes. Analogously we have evaluated integrals of series by exchanging the integration and summation processes. Review these manipulations and provide rigorous verifications of their validity.

4.- Show that

(a) The power series expansion about the origin of an even analytic function contains only even powers of z.

(b) If $f(z)$ is entire and $|f|/|z|^n$ is bounded as $|z| \to \infty$ (where n is some positive integer) then $f(z)$ must be a polynomial of degree $\leq n$.

Due October 17 at noon.